PHYSICAL REVIEW E **79**, 015301(R) (2009)

Evolution of the acceleration field and a reformulation of the sweeping decorrelation hypothesis in two-dimensional turbulence

F. Schwander*
Department of Aeronautics, Imperial College, London SW7 2AZ, United Kingdom

E. Hascoët and J. C. Vassilicos†
Department of Aeronautics and Institute for Mathematical Sciences, Imperial College, London SW7 2AZ, United Kingdom

(Received 26 June 2008; revised manuscript received 1 September 2008; published 22 January 2009)

From simulations of two-dimensional inverse energy cascading turbulence, we show that points with low acceleration values are predominantly advected by the local fluid velocity. The fluid velocity $\mathbf{u}$ in the global frame and the fluid velocity $\boldsymbol{\xi}$ in the frame moving with a low-acceleration point are approximately statistically independent. This property remains valid in high-acceleration regions but only in the direction of the local acceleration vector. In the perpendicular direction, the acceleration velocity $\mathbf{V}_a=\mathbf{u}-\boldsymbol{\xi}$ is approximately independent of $\boldsymbol{\xi}$ everywhere. These statistical independences constitute our formulation of the sweeping decorrelation hypothesis for two-dimensional inverse energy cascading turbulence.

DOI: 10.1103/PhysRevE.79.015301 PACS number(s): 47.27.Gs

Sweeping, the advection of small eddies by eddies of larger size, is a crucial mechanism in turbulent flows. It has been shown to dominate the variance of the Eulerian time derivative of the velocity, as it contributes more to it than the fluid acceleration at high Reynolds numbers [1]. This dominance of sweeping implies a degree of persistence in the local frame of reference moving with the fluid, so that the velocity vector approximately follows the trajectory of a fluid element [2]. It has been suggested that, similarly, sweeping dominates the Eulerian temporal fluctuations of the acceleration itself, with the corollary that the acceleration is nearly constant along most fluid trajectories and that the velocity with which the acceleration moves in a turbulent flow is approximately equal to the local fluid velocity [3]. Several variables were introduced to quantify the sweeping of zero-acceleration points, the first being the acceleration velocity $\mathbf{V}_a=d\mathbf{s}(t)/dt$, where $\mathbf{s}(t)$ is the position of a point that moves so that the acceleration on it is constant in time $\{d\mathbf{a}[\mathbf{s}(t),t]/dt=0\}$. The second such quantity is the difference between the flow velocity and the acceleration velocity, denoted by $\boldsymbol{\xi}=\mathbf{u}-\mathbf{V}_a$. That $\mathbf{a}=\mathbf{0}$ points are swept, on average, by the fluid follows from Kolmogorov scaling applied to $\langle\boldsymbol{\xi}^2|\mathbf{a}=\mathbf{0}\rangle$ which, as recently shown [3], gives

$$\langle\boldsymbol{\xi}^2|\mathbf{a}=0\rangle \propto u'^2(\mathcal{L}/\eta)^{-2/3}, \qquad (1)$$

in terms of the rms fluid turbulence velocity u' and the outer and inner length scales of the homogeneous isotropic turbulence $\mathcal{L}$ and η. The resulting asymptotic statistical correspondence between $\mathbf{V}_a$ and $\mathbf{u}$ in the limit where the Reynolds number $\mathrm{Re}\sim(\mathcal{L}/\eta)^{3/4}$ tends to infinity establishes the property that zero-acceleration points are predominantly swept by the fluid velocity in high-Re turbulence. This matters as the sweeping property of much of the acceleration field underpins the "sweep-stick" mechanism recently introduced to explain the preferential concentration of small and heavy inertial particles in homogeneous isotropic turbulence [3–5]. In terms of $\langle\boldsymbol{\xi}^2|\mathbf{a}=\mathbf{0}\rangle$ only, this result remains rather weak. A much stronger statement than Eq. (1) would be in the coincidence of the probability distribution functions (PDFs) of the acceleration and fluid velocities and in the form of the joint PDFs. The present study aims at refining the characterization of the velocity at which points with zero, as well as finite, acceleration travel in a turbulent flow, by investigating conditional PDFs of the acceleration velocity and joint PDFs of the fluid and acceleration velocities in numerical simulations of two-dimensional, inverse cascading turbulence.

The numerical procedure is similar to that described in [4], and we simply recall its main features. A two-dimensional stationary, homogeneous, isotropic turbulent velocity field, with a well-defined inertial range in which energy is transferred from small to large scales, is established and sustained by holding the entropy constant at high wave numbers in a shell around k_f. The kinetic energy wave number spectrum in the inertial range has a power-law dependence with a $-5/3$ exponent, as expected from the inverse cascade regime in the flow [6–8]. Since our interest is in the turbulence dynamics, forcing scales are removed by low-pass filtering of the velocity field in wave number space. The inner cutoff wavelength is then defined as $\eta=2\pi/k_c$ where $k_c<k_f$ is the cutoff wave number. The spatial resolution in the simulation is moderate, with a grid using 512^2 points, which allows one to reach a maximum scale separation $\mathcal{L}/\eta\simeq 12$. The acceleration field is computed exactly, via its Fourier transform $\hat{a}_i(\mathbf{k})$, as a function of the velocity field:

$$\hat{a}_l(\mathbf{k}) = i\sum_{m,n}\frac{k_l k_m k_n}{\mathbf{k}^2}\widehat{u_m u_n}(\mathbf{k}) - D(\mathbf{k})\hat{u}_l(\mathbf{k}), \qquad (2)$$

where $\hat{b}(\mathbf{k})$ denotes the Fourier transform of the field b, k_i and u_i the components of the wave vector and velocity vector, and $D(\mathbf{k})$ is a generalized dissipation operator [4]. The time derivative of the acceleration, required to compute the acceleration velocity, is obtained in a similar manner. The scaling of the acceleration variance, established by varying the cutoff wave number, follows the scaling $\langle\mathbf{a}^2\rangle^{1/2}$

*frederic.schwander03@imperial.ac.uk
†j.c.vassilicos@imperial.ac.uk

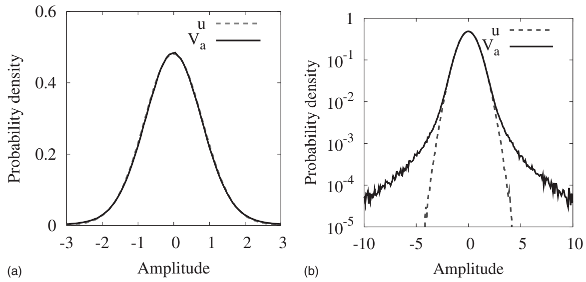

FIG. 1. PDFs of the fluid and acceleration velocities normalized by u' on points with zero acceleration in linear (left) and semilogarithmic (right) representations.

$\propto u'^2\mathcal{L}^{-1}(\mathcal{L}/\eta)^{1/3}$. It shows that our filtering procedure preserves the small-scale character of the acceleration, since the Kolmogorov scaling [9] relies on the latter. We also confirm the Kolmogorov-type scaling of $\langle\boldsymbol{\xi}^2|\mathbf{a}=0\rangle$ given by Eq. (1).

Owing to the isotropy of the flow, the unconditional velocity statistics are invariant to rotation of the direction along which the velocity vector is projected, and this also applies to statistics taken on points with zero acceleration, so that the statistics of only one component of velocity vectors is reported here.

Figure 1 (left) shows that, even for the moderate scale separation reached in this simulation, the conditional fluid and acceleration velocity distributions are almost indistinguishable for amplitudes up to $3u'$, and are both nearly Gaussian. Closer investigation of both PDFs in a semilogarithmic representation [Fig. 1 (right)] shows that the agreement is incomplete and breaks down at amplitudes larger than $2u'$ where the acceleration velocity has long tails, which contrast with the Gaussian fall-off of the velocity distribution. At these values, the acceleration velocity is much larger than any reasonable local fluid velocity and in all likelihood, events constituting the tails of the distribution are exceptional (though not unimportant) dynamical events where the acceleration changes rapidly along a fluid element trajectory, in which case sweeping is irrelevant. These events, however, are rare, with a probability density not exceeding 1/100, so that the PDFs of the acceleration velocity and the fluid velocity are nearly the same except for high amplitudes.

It is legitimate to question whether the near equality of both distributions is not accidental but also applies locally, as required in order to prove the sweeping of zero-acceleration points with the fluid. We therefore examine the joint PDFs of the velocities, shown in Fig. 2, with the aim of discarding a possible statistical independence between the acceleration velocity and the fluid velocity that would exist if the local, statistical equality was not verified. This can be done directly since, given the symmetry of the individual velocity PDFs, if a couple of velocities are statistically independent, their joint probability must be an even function of its arguments.

TABLE I. Velocity statistics conditional on the acceleration.

$\lvert\mathbf{a}\rvert$	$\langle\mathbf{u}_\parallel^2\rangle$	$\langle\mathbf{u}_\perp^2\rangle$	$\langle\mathbf{u}_\parallel^4\rangle/\langle\mathbf{u}_\parallel^2\rangle^2$	$\langle\mathbf{u}_\perp^4\rangle/\langle\mathbf{u}_\perp^2\rangle^2$
0	0.708	0.706	3.174	3.200
$a'/2$	0.836	0.947	3.065	2.831
a'	0.883	1.290	3.069	2.482
$2a'$	0.831	2.163	3.266	1.982
$4a'$	0.703	3.615	3.184	1.616
Global	0.826	1.171	3.027	2.690

Formally, none of the three joint PDFs shown in Fig. 2 exactly satisfies the symmetry properties required by statistical independence. The left plot of Fig. 2 indicates that the strongest statistical dependence occurs between the acceleration velocity and the fluid velocity, whose joint PDF [Fig. 2 (right)] can be approximated as $p(\mathbf{V}_a,\mathbf{u})=f(\mathbf{u})g(\mathbf{u}-\mathbf{V}_a)$ because $\mathbf{u}$ and $\boldsymbol{\xi}$ seem to be the least statistically dependent of the variables plotted in Fig. 2. The crucial point is that the PDF $g(\boldsymbol{\xi})$ has its maximum at $\boldsymbol{\xi}=\mathbf{u}-\mathbf{V}_a=0$ and hence, for a given fluid velocity $\mathbf{u}$, the likeliest value taken by the acceleration velocity at zero-acceleration points is equal to $\mathbf{u}$. This observation and Eq. (1) establish statistically the sweeping of zero-acceleration points by the local fluid velocity.

We now investigate the degree to which this result can be extended to points with finite acceleration by considering statistics of the three velocities conditional on $|\mathbf{a}|=\{a'/2,a',2a',4a'\}$, where a' is the global rms value of the acceleration. These statistics are no longer invariant by rotation and must be considered separately in the directions along the acceleration vector (longitudinal direction, subscript $\parallel$) and normal to it (transverse direction, with subscript $\perp$). The differences between both velocity components are evident in Table I, where it is shown that the conditional velocity variance has a maximum for $|\mathbf{a}|=a'$ in the longitudinal direction, while it is a monotonically increasing function of $|\mathbf{a}|/a'$ for the sampled acceleration amplitudes in the transverse direction. Furthermore, the PDFs differ in shape depending on the orientation relative to the acceleration vector since conditional flatnesses (see Table I) take values neighboring 3 for all accelerations in the longitudinal direction, suggesting that the corresponding velocity PDFs are near Gaussian, but decrease with acceleration in the transverse direction, indicating progressive departure from normality in this direction.

Our concern here is how the fluid velocity statistics compare with the statistics of the acceleration velocity and,

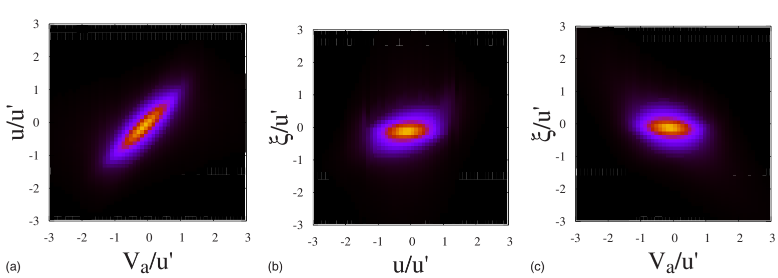

FIG. 2. (Color online) Joint PDFs $p(\mathbf{V}_a,\mathbf{u})$ (left), $p(\mathbf{u},\boldsymbol{\xi})$ (middle), and $p(\mathbf{V}_a,\boldsymbol{\xi})$ (right) on points with zero acceleration.

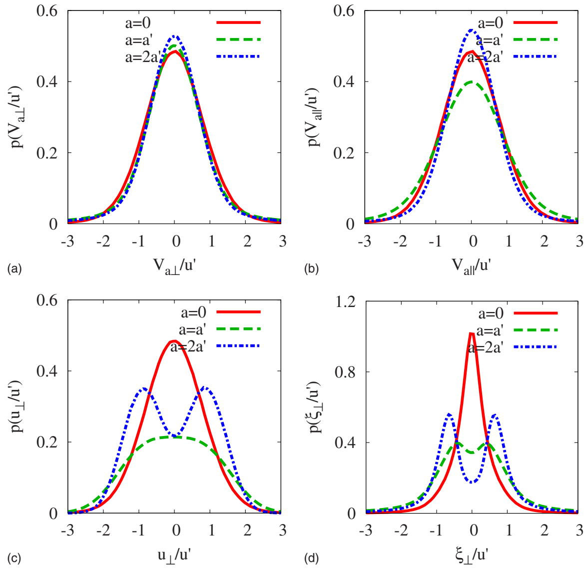

FIG. 3. (Color online) PDFs of the transverse component of the fluid velocity (top left), the longitudinal acceleration velocity (top right), the transverse fluid velocity (bottom left), and the transverse component of $\boldsymbol{\xi}$ (bottom right) for $|\mathbf{a}|=\{0,a',2a'\}$.

should points with finite acceleration be swept with the fluid, we would expect both velocity distributions to be similar regardless of the direction relative to the local acceleration vector. Direct comparison of the distributions through their sole moments is prevented by the tails of the acceleration velocity distribution already evoked for zero acceleration points, which contribute greatly to the variance of the distribution and lead to divergence of the fourth-order moment. These tails, not shared by the velocity PDFs, further imply that correspondence between the velocity distributions can occur only at moderate velocity, and this limited correspondence can only be assessed by consideration of the distributions themselves.

Figure 3 shows the conditional PDFs for velocities up to $3u'$. In the longitudinal direction, both $\mathbf{u}$ and $\mathbf{V}_a$ have PDFs close to Gaussian for velocities lower than $2u'$, but their variances vary with acceleration. For $\mathbf{V}_a$, this variation results from changes in both the tails and the Gaussian part of the PDF. More pronounced differences are visible in the transverse direction where, in contrast with the acceleration velocity PDF (Fig. 3, top left), which depends only weakly on the acceleration and mirrors the PDF already given for zero-acceleration points, the other two PDFs (for the fluid velocity $\mathbf{u}$ and for $\boldsymbol{\xi}$, in Fig. 3, bottom left and right) vary in shape with the acceleration amplitude and progressively broaden. This broadening is negligible when the acceleration is smaller than a' but becomes significant for $|\mathbf{a}|\geqslant a'$ when both PDFs become bimodal, likely as a result of the coexistence of identically distributed left- and right-handed intense vortices in the turbulence. This aspect is particularly marked in the PDF of the transverse difference velocity $\boldsymbol{\xi}$. It here appears that the fluid and acceleration velocities differ qualitatively and systematically on points where the acceleration is greater than or comparable to a'. The statistical correspondence between $\mathbf{V}_a$ and $\mathbf{u}$ observed for zero-acceleration points therefore remains a good approximation for low-acceleration points but cannot be generalized to points with acceleration larger than a'.

We now consider joint PDFs conditional on acceleration values, both high and low, e.g., as in Fig. 4, where we report results for acceleration $|\mathbf{a}|=4a'$. These joint PDFs are distributed around a single peak in the longitudinal direction but exhibit two distinct lobes in the transverse direction. These observations are valid for all acceleration values. The two PDF lobes in the transverse direction are very close to each other, and therefore nearly indistinguishable, at small acceleration values, but become clearly distinct as the acceleration magnitude increases above a'. Results such as Fig. 4, which we have obtained for different conditional accelerations suggest the following two statistical decorrelations irrespective of the conditional acceleration value: (1) the velocity $\boldsymbol{\xi}$ and the fluid velocity are statistically independent in the longitudinal direction, and (2) the acceleration velocity and the velocity $\boldsymbol{\xi}$ are statistically independent in the transverse direction.

These hypotheses enable us to write the joint PDFs as functions of the single velocity PDFs:

$$p(\mathbf{V}_{a\|},\boldsymbol{\xi}_{\|}) = p(\boldsymbol{\xi}_{\|})p_{\mathbf{u}_{\|}}(\mathbf{V}_{a\|}+\boldsymbol{\xi}_{\|}), \tag{3}$$

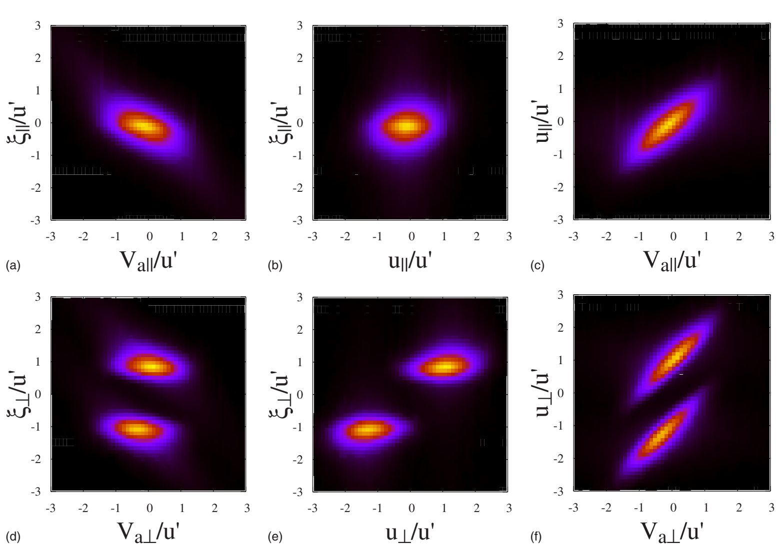

FIG. 4. (Color online) Joint PDFs sampled on points with $|\mathbf{a}|=4a'$: $p(\mathbf{V}_a,\boldsymbol{\xi})$ (left), $p(\mathbf{u},\boldsymbol{\xi})$ (middle), and $p(\mathbf{V}_a,\mathbf{u})$ (right) in the longitudinal (upper row) and the transverse (lower row) direction.

RAPID COMMUNICATIONS

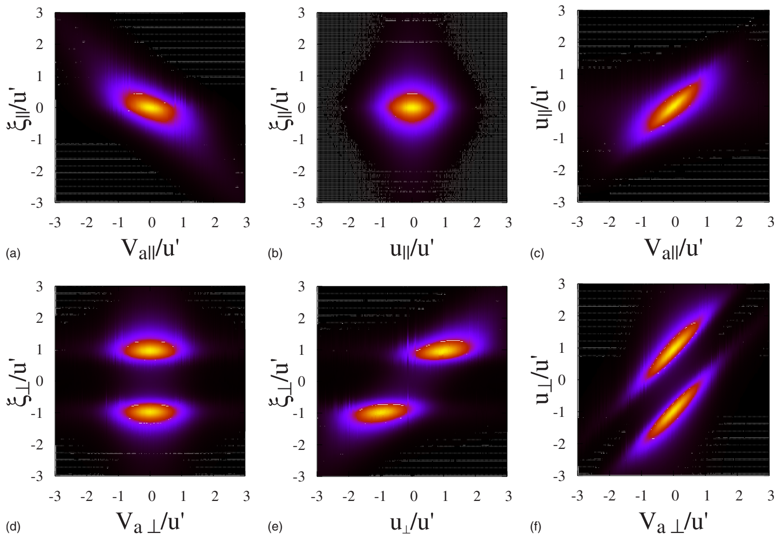

FIG. 5. (Color online) Joint PDFs, computed using the single-velocity PDFs in conjunction with Eqs. (3)–(6) on points with $|\mathbf{a}|=4a'$: $p(\mathbf{V}_a,\boldsymbol{\xi})$ (left), $p(\mathbf{u},\boldsymbol{\xi})$ (middle), and $p(\mathbf{V}_a,\mathbf{u})$ (right) in the longitudinal (upper row) and transverse (lower row) directions.

$$p(\mathbf{V}_{a\parallel},\mathbf{u}_{\parallel})=p(\mathbf{u}_{\parallel})p_{\boldsymbol{\xi}_{\parallel}}(\mathbf{u}_{\parallel}-\mathbf{V}_{a}\parallel), \quad (4)$$

$$p(\mathbf{u}_{\perp},\boldsymbol{\xi}_{\perp})=p(\boldsymbol{\xi}_{\perp})p_{\mathbf{V}_{a\perp}}(\mathbf{u}_{\perp}-\boldsymbol{\xi}_{\perp}), \quad (5)$$

$$p(\mathbf{V}_{a\perp},\mathbf{u}_{\perp})=p(\mathbf{V}_{a\perp})p_{\boldsymbol{\xi}_{\perp}}(\mathbf{u}_{\perp}-\mathbf{V}_{a\perp}). \quad (6)$$

The joint PDFs obtained using Eqs. (3)–(6) are plotted in Fig. 5, and despite the lack of exact symmetry of the joint PDFs $p(\mathbf{u}_{\parallel},\boldsymbol{\xi}_{\parallel})$ and $p(\mathbf{V}_{a\perp},\boldsymbol{\xi}_{\perp})$ necessary for exact statistical independence to apply, there is good qualitative agreement with the actual joint PDFs, thus supporting the observed approximate statistical independences. The approximate statistical independence in the transverse direction is especially interesting because it is reminiscent of the sweeping decorrelation hypothesis used in [10], this time expressed as an approximate decorrelation between the transverse acceleration velocity $\mathbf{V}_a$ and the transverse fluid velocity in the local frame moving with this velocity $\boldsymbol{\xi}$. It is noteworthy that the type of "sweeping decorrelation" which we observe here is approximately valid for all acceleration levels in the transverse but not in the longitudinal direction where it is the fluid velocity $\mathbf{u}$ in the global frame of reference, which is approximately decorrelated with $\boldsymbol{\xi}$. However, where the acceleration is small compared to a', $\mathbf{V}_a \approx \mathbf{u}$. It is therefore fair to say that in those regions of low acceleration the relative velocity $\boldsymbol{\xi}$ is independent of the acceleration velocity $\mathbf{V}_a$ that sweeps the local acceleration field, irrespective of the transverse or longitudinal orientation. This reformulated sweeping decorrelation breaks down where the acceleration is comparable to or above a' but only in the longitudinal direction, and it is interesting, and perhaps even unexpected, that it survives at such high accelerations in the transverse direction.

Our reformulated sweeping decorrelation property implies that the turbulent fluid velocity $\mathbf{u}$ is a sum of two velocities $\mathbf{V}_a$ and $\boldsymbol{\xi}$, which are approximately decorrelated where the acceleration is relatively weak and approximately decorrelated in the transverse direction everywhere (irrespective of whether the local acceleration is weak or high). The statistics of these two velocities depend differently on the acceleration. As shown in Fig. 3, the PDF of $\mathbf{V}_a$ retains the same, approximate Gaussian shape at all acceleration values but predominantly changes variance with conditional acceleration level. The same figure shows that the dependence of the PDF of the difference velocity $\boldsymbol{\xi}$ is more dramatic, involving a clear move from an apparently single-lobe to a clearly double-lobe structure as conditional acceleration level increases. The qualitative shape of the transverse turbulence fluid velocity's PDF and the acceleration dependence of this shape are dominated by the transverse difference $\boldsymbol{\xi}$.

In summary, this work on acceleration motion in two-dimensional, inverse cascading turbulence shows that points with low acceleration are predominantly swept with the fluid. For large accelerations, the acceleration velocity $\mathbf{V}_a$ and fluid velocity $\mathbf{u}$ differ statistically, as illustrated by their respective Gaussian and non-Gaussian distributions in the direction perpendicular to the acceleration, and sweeping therefore no longer applies. Nevertheless, approximate statistical independence is observed for all accelerations: between the longitudinal projections of the fluid velocity $\mathbf{u}$ in the global frame and the fluid velocity $\boldsymbol{\xi}$ in the frame moving with the local acceleration, and between the transverse projections of $\boldsymbol{\xi}$ and $\mathbf{V}_a$. These statistical properties constitute our formulation of the sweeping decorrelation hypothesis for two-dimensional inverse energy cascading turbulence.

[1] A. Tsinober *et al.*, Phys. Fluids **13**, 1974 (2001).
[2] S. Goto *et al.*, Phys. Rev. E **71**, 015301(R) (2005).
[3] L. Chen *et al.*, J. Fluid Mech. **553**, 143 (2006).
[4] S. Goto and J. C. Vassilicos, New J. Phys. **6**, 65 (2004).
[5] S. Goto and J. C. Vassilicos, Phys. Rev. Lett. **100**, 054503 (2008).
[6] R. H. Kraichnan, Phys. Fluids **10**, 1417 (1967).
[7] C. E. Leith, Phys. Fluids **11**, 671 (1968).
[8] G. K. Batchelor, Phys. Fluids **12**, 233 (1969).
[9] W. Heisenberg, Z. Phys. **124**, 628 (1948).
[10] H. Tennekes, J. Fluid Mech. **67**, 561 (1975).